The Multi-Dam System of the Zambezi River Basin

This paper establishes multi-dam system \texttt{model(6.2)} and multi-objective optimization \texttt{model (6.4)} to decide the location of dams and provide strategies of modulating the water flow under the regular situation and emergency situation.

In the requirement 1, we give a brief assessment of the three options, with sufficient details to provide costs, generating capacity and population of people benefit from the Zambezi River Basin associated with each option. We gain the result in the \texttt{Figure 3}.

In the requirement 2, firstly, we set up single dam system \texttt{model(6.1)}. We gain maximum storage of each dam and make water inflow/outflow per unit time as design variables. According to water balance principle, we gain the model of describing water inflow/outflow of dam. We can figure out outflow curve with the local hydrologic data.

Secondly, we establish the simple multi-dam system model. The loss of water flow between dams is ignored. Then we gain the model of describing water flow between the \texttt{i-th} dam and the \texttt{(i+1)th} dam. Then we take water flow of tributaries of Zambezi River and water evaporation into consideration. We design a variable \( \sigma_i^2 \) which describes the variation of the flow between the dams and add \( \Delta_i \) into multi-dam system \texttt{model(6.2)} and make model as close as possible to the reality.

Then, we establish a multi-objective optimization \texttt{model(6.4)}. Based on the Three-Dimensional topography of Zambezi River and requirement of geographical condition of building dam, we screen the several possible location of dams. According to equation [7] and equation [2], we obtain the function of costs and function of generating capacity respectively. We figure out the variance \( \sigma_i^2 \) of water flow in the multi-dam system and average water flow of local hydrologic data. Variance \( \sigma_i^2 \) will decide the safety of the dam. In conclusion, we obtain a multi-objective optimization \texttt{model(6.4)}. To solve this model, we transform the multiple targets into single target by using the weight methods. We apply the multi-level optimization search method to solve the model, and get the locations of dams.

Finally, we provide strategies of modulating the water flow under the regular situation and emergency situation \texttt{Table 2}.

\textbf{Key Words}: Multi-Dam System, Multi-Objective Optimization Model, Water Balance Principle, Strategy of Water Modulating, Multi-Level Optimization Search Method
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1 Introduction

1.1 Background

The Zambezi River Basin (ZRB) is shared by eight countries, Tanzania and Namibia joining those quoted above. In addition to meeting the basic needs of more than 30 million people and playing a central role in the riparian economies, the river needs be developed by ZRA. [8]

420 kilometers downstream from Victoria Falls, the Kariba Dam is the largest man-made reservoir in the world. At a height of 128m and with a crest length of 617m, the dam has the capacity of holding 181 billion cubic meters of water. Due to, however, lacking of maintenance and geographical condition, the Kariba Dam is facing serious safety issues.

![The Kariba Dam](image)

Figure 1 The Kariba Dam

In this paper, we estimate the generating capacity, capacity of water modulating and maintenance costs of the existed Kariba Dam. Aimed to improve Zambezi river’s economic efficiency, we design multi-dam system which increases generating capacity greatly and prevent population of Zambezi river region from flood. Beside we provide a series of strategies of water modulating to the ZRA managers.

1.2 Our work

We give a brief assessment of the three options listed, with sufficient detail to provide costs, generating capacity and population of people benefit from Zambezi River associated with each option.
Our work begins with single dam model. In this model, we use water balance principle to describe water inflow/outflow of a dam. Based on this, ignoring the increase or decrease of water flow between ith dam and (i+1)th dam, we obtain basic multiple dam system.

After that, we put the influence of tributaries, evaporation, rainfall and other water cycles into consideration to make our multi-dam system model as close as possible to the reality. By approximating to the hydrological data of the Zambezi River Basin with piecewise linear interpolation method. Besides, in order to balance safety and costs, we finally obtain a multi-objective optimization model by calculating the variance of the downstream water flow and estimating the costs of dams.

Based on the actual situation of the Zambezi River Basin, we firstly preliminary screening the possible locations of building dams, then we use multi-level optimization search method to solve our model (6.4) with normal water flow distribution.

2 Problem restatement

- Provide an assessment of three options with their potential costs and benefits.
- Develop a Multi-Dam System which balances between safety and cost reasonably and this new system will have a greater capacity of water management than the existing Kariba Dam.
- Provide strategies of water modulating in the regular situation and extreme situation under the new system.

3 Terminology

3.1 Terms

![Figure 2 The dam diagram](image)
3.2 Symbols

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_t^i$</td>
<td>The water-storage of the $i$th dam at time $t$</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$Q_0^i$</td>
<td>The initial water-storage of the $i$th dam</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$Q_{max}^i$</td>
<td>The maximum water-storage of the $i$th dam</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$Q_K$</td>
<td>The maximum water-storage of Kariba dam</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$q_{in}^i$</td>
<td>The water inflow per unit time of the $i$th dam</td>
<td>$m^3/s$</td>
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<tr>
<td>$q_{out}^i$</td>
<td>The water outflow per unit time of the $i$th dam</td>
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<td>$q_{max}^i$</td>
<td>The maximum water outflow per unit time of the $i$th dam</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$q_{AVG}^i$</td>
<td>The average downstream water flow of the $i$th dam</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$T$</td>
<td>A cycle</td>
<td>year</td>
</tr>
<tr>
<td>$\sigma_i^2$</td>
<td>The variance of downstream water flow per unit time of the $i$th dam</td>
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</tr>
<tr>
<td>$C_d^i$</td>
<td>The cost estimation of the $i$th dam</td>
<td>$$$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The coefficient of inflation</td>
<td></td>
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<tr>
<td>$K^i$</td>
<td>The scale factor of the $i$th dam</td>
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<tr>
<td>$V^i$</td>
<td>The installed capacity of the $i$th dam</td>
<td>$kW$</td>
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<tr>
<td>$H^i$</td>
<td>The height of the $i$th dam</td>
<td>m</td>
</tr>
<tr>
<td>$h_t^i$</td>
<td>The water head of the $i$th dam at time $t$</td>
<td>m</td>
</tr>
<tr>
<td>$p^i$</td>
<td>The hydroelectricity of the $i$th dam</td>
<td>$kW$</td>
</tr>
<tr>
<td>$\eta^i$</td>
<td>The dimensionless efficiency of the turbine of the $i$th dam</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>The density of water</td>
<td>$kg/m^3$</td>
</tr>
<tr>
<td>$P_K$</td>
<td>The hydroelectricity of Kariba dam</td>
<td>$MW$</td>
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<tr>
<td>$\Delta^i$</td>
<td>The variation of downstream water flow of the $i$th dam</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>The weight factor</td>
<td></td>
</tr>
<tr>
<td>$\lambda^i$</td>
<td>The governing equation coefficient of the $i$th dam</td>
<td></td>
</tr>
</tbody>
</table>

4 General assumptions

- Assume the reservoir capacity of the dam is constant.
- Ignore the costs of maintaining dam and generating devices after the dam has been built.
- Assume the data we used is accurate.
- Geological changes like debris flow will not affect the river.
5 Requirement 1

Designed as a double curvature concrete arc dam, the Kariba Dam was constructed across the Zambezi River between 1956 and 1959. Nowadays, the Kariba Dam is central to energy safety in the form of hydropower electricity generated for Zimbabwe and Zambia and as the Zambezi continues to flow towards Mozambique it flows into the Cahora Bassa Lake and then continues to provide electricity for Mozambique and South Africa. However, lacking of maintenance and geographical condition, the Kariba Dam is facing serious safe issue. Therefore, we give three options to solve problems and assessments of options.

5.1 Assessment of option1: Repairing the existing Kariba Dam

The existing Kariba Dam are currently in danger of breaking and generating capacity of dam can’t meet the basic requirement of electric energy of people. Besides being repaired, the Kariba Dam will be added the 660MW installed electricity capacity and it’s maximum generating capacity will reach 2350MW. The total costs will be 598 million dollars.

5.2 Assessment of option2: Rebuilding the existing Kariba Dam

Due to the huge scale of existing dam, it will cost about 3 years to remove the dam. To build the same scale dam, the Zambian government and the Zimbabwean government will undertake the high costs which reaches up to 5 billion dollar within 5 years. During the reconstruction of the dam, the per capita electricity consumption countries in the Zambezi River Basin will be decreased seriously.

5.3 Assessment of option3: Removing the Kariba Dam and replacing it with a series of ten to twenty smaller dams along the Zambezi River

According to the multi-objective optimization model (6.4), setting up a multi-dam system will cost 13.09 billion dollars. This system will meet electricity demand of population of region and improve capacity of water modulating greatly.

5.4 General Assessment
Obviously, option 3 is the better choice.

6 Requirement 2

In the requirement 2, we build single-dam system which concludes maximum storage of dam and the distribution of water inflow/outflow per unit time and costs. Based on this system, we figure out the mathematical relationship between each single dam system. Regarding the costs of building dam system and safety of the dam system as restricted factors, we obtain the basic multi-dam system. Then we take water flow distribution of tributaries of the Zambezi River and water evaporation into consideration to make model as close as possible to reality. We get the multi-dam system model (6.2).

Based on the three-dimensional topography of the Zambezi River and requirement of geographical condition of building dam, we screen the several possible location of dams Figure 8,9,10 and estimate the maximum storage of each dam. Thus we obtain a multi-objective optimization model (6.4) which is changed into single-objective optimization model (6.4.3) by using weight methods. This model (6.4.3) can give ZRA the possible location of dams, and strategies of water modulating in the regular/irregular situation.

6.1 Single-Dam System Model

To simplify the real dam system, we ignore water evaporation, leakage and water flow of tributaries of the Zambezi River Basin. After that we build a simple dam model, as shown in the Figure 4:
According to the Water Balance Principle, we figure out water-storage at time $t$:

$$Q_t = Q_0 + \int_0^t q_{in} - q_{out} \, dt$$

Where:
- $Q_0$ is the initial water-storage of the dam
- $Q_t$ is the water-storage of the dam at time $t$
- $q_{in}$ is the water inflow per unit time of the dam
- $q_{out}$ is the water outflow per unit time of the dam

Because the maximum discharging of dam is $q_{max}$, we get a constraint condition below:

$$s.t. \ 0 \leq q_{out} \leq q_{max}$$

Water-storage of dam should be less than design flood level $Q_{max}$, so another constraint condition is:

$$Q_0 \leq Q_t \leq Q_{max}$$

Then, calculate the amount of available power:

$$P = \eta q_{out} g h_t$$

Where:
- $P$ is the hydroelectricity of the dam
- $\eta$ is the dimensionless efficiency of the turbine of the dam
- $\rho$ is the density of water
- $g$ is the acceleration due to gravity
- $h_t$ is the water head at time $t$

According to the empirical formula [7], the cost of building a dam is:

$$C_d = \alpha \cdot 0.34 K^{0.9} \left[ \frac{V}{(H)^{0.3}} \right]^{0.74}$$

Where,
- $C_d$ is the cost estimation of the dam
- $\alpha$ is the coefficient of inflation [6]
- $K$ is the scale factor of the dam (large-scale $K = 7.7 \times 10^4$, small-scale $K = 5 \times 10^4$)
- $V$ is the installed capacity of the dam

We can describe inflow/outflow situation of single-dam by combining the equations above. Finally we obtain the Single-Dam System Model:
\[ Q_t = Q_0 + \int_0^t (q_{in} - q_{out}) \, dt \]
\[ 0 \leq q_{out} \leq q_{max} \]
\[ Q_0 \leq Q_t \leq Q_{max} \]
\[ P = \eta q_{out} g h_t \]
\[ C_d = \alpha \cdot 0.34K^{0.9} \left( \frac{V}{(H^{0.3})} \right)^{0.74} \]

Where:

- \( Q_0 \) is the initial water-storage of the dam
- \( Q_t \) is the water-storage of the dam at time \( t \)
- \( q_{in} \) is the water inflow per unit time of the dam
- \( q_{out} \) is the water outflow per unit time of the dam
- \( q_{max} \) is the maximum water outflow per unit time of the dam
- \( Q_{max} \) is the maximum water-storage of the dam
- \( P \) is the hydroelectricity of the dam
- \( \eta \) is the dimensionless efficiency of the turbine of the dam
- \( \rho \) is the density of water
- \( g \) is the acceleration due to gravity
- \( h_t \) is the water head at time \( t \)

### 6.2 Multi-Dam System Model

Ignoring influence made by water evaporation, leakage and water flow of tributaries of the Zambezi River Basin, we assume the outflow per unit time of the \( i \)th dam is equal to the inflow per unit time of the \((i+1)\)th dam,

\[ q_{in}^{i+1} = q_{out}^i \]

Where:

- \( q_{in}^{i+1} \) is the inflow per unit time of the \((i+1)\)th dam
- \( q_{out}^i \) is the outflow per unit time of the \( i \)th dam
Besides, the storage capacity of Multi-Dam System should be greater than the Kariba Dam, thus

\[ \sum_{i=1}^{n} Q_i^t \geq Q_K \]

Where:
- \( Q_i^t \) is the water-storage of the \( i \)-th dam at time \( t \)
- \( Q_K \) is the maximum water-storage of Kariba dam

Generating capacity of the Multi-Dam System should also be greater than the Kariba Dam,

\[ \sum_{i=1}^{n} P_i \geq P_K \]

Where:
- \( P_i \) is the hydroelectricity of the \( i \)-th dam
- \( P_K \) is the hydroelectricity of Kariba dam

Therefore, based on the model (6.1), we gain the Multi-Dam System Model as below,

\[
\begin{align*}
Q_i^t &= Q_0^i + \int_0^t q_{in}^i - q_{out}^i \, dt \\
C_d^i &= \alpha \cdot 0.34K_i^{0.9} \left[ \frac{V^i}{H_i^{0.3}} \right]^{0.74} \\
P_i &= \eta_i \rho q_{out}^i g h_t^i \\
q_{out}^{i+1} &= q_{out}^i \\
0 \leq q_{out}^i \leq q_{max}^i \\
Q_0^i \leq Q_i^t \leq Q_{max}^i \\
\sum_{i=1}^{n} P_i \geq P_K \\
\sum_{i=1}^{n} Q_i^t \geq Q_K
\end{align*}
\]

### 6.3 Take Factors of Variation of Water Flow into Consideration
We take water flow of tributaries of Zambezi River Basin and water evaporation into consideration to make the model (6.2) as close as possible to reality, we get an error correction function:

\[ q_{in}^{i+1} = q_{out}^i \pm \Delta^i \]

Where:

\( \Delta^i \) is the variation of downstream water flow of the ith dam

Based on the model (6.2), we put \( \Delta^i \) into it. The Multi-Dam System Model is gained:

\[
\begin{align*}
Q_t^i &= Q_0^i + \int_0^t q_{in}^i - q_{out}^i \, dt \\
C_d^i &= \alpha \cdot 0.34K^{i0.9} \left[ \frac{\nu^i}{(H^i)^{0.3}} \right]^{0.74} \\
p_i &= \eta^i p q_{out}^i g h_t^i \\
q_{in}^{i+1} &= q_{out}^i \pm \Delta^i \\
0 &\leq q_{out}^i \leq q_{\text{max}}^i \\
Q_0^i &\leq Q_t^i \leq Q_{\text{max}}^i \\
\sum_{i=1}^n p_i &\geq P_K \\
\sum_{i=1}^n Q_t^i &\geq Q_K
\end{align*}
\]

6.4 Multi-Objective Optimization Model

6.4.1 The Construction Costs of Dams

According to Formula [7], we can use the function to describe the costs of each dam. Furthermore, we want to make the total construction costs of Multi-Dam System as low as possible, which means:
\[
\min C_d = \sum_{i=1}^{n} C_d^i
\]

Where:

\(C_d\) is the total costs of the \(n\) dams

### 6.4.2 The Estimation of Dam's Safety

Take the safety of dam system into consideration, we figure out the variance \(\sigma_i^2\) of water flow in the Multi-Dam System and average water flow of local hydrologic data. Variance \(\sigma_i^2\) will decide the safety of the dam. Thus, the second target is rendering variance \(\sigma_i^2\) as small as possible, which means:

\[
\min \sigma_i^2 = \frac{1}{T} \int_0^T (q_{out}^i - q_{AVG}^i)^2 \, dt
\]

Where:

\(\sigma_i^2\) is the variance of downstream water flow per unit time of the \(i\)th dam

\(T\) is a year

\(q_{AVG}^i\) is the average downstream water flow of the \(i\)th dam

Finally, we establish a Multi-Objective Optimization Model:

\[
\begin{align*}
\min \sigma_i^2 &= \frac{1}{T} \int_0^T (q_{out}^i - q_{AVG}^i)^2 \, dt \\
\min C_d &= \sum_{i=1}^{n} C_d^i \\
Q_t^i &= Q_0^i + \int_0^t q_{in}^i - q_{out}^i \, dt \\
C_d^i &= \alpha \cdot 0.34K_i^{0.9} \left[ \frac{V_i^{0.9}}{(H_i^{0.3})} \right]^{0.74} \\
s.t. \begin{cases}
p^i = n^i p q_{out}^i gh t^i \\
q_{in}^{i+1} = q_{out}^i \pm \Delta^i \\
0 \leq q_{out}^i \leq q_{max}^i \\
Q_0^i \leq Q_t^i \leq Q_{max}^i \\
\sum_{i=1}^{n} p^i \geq P_K \\
\sum_{i=1}^{n} Q_t^i \geq Q_K
\end{cases}
\end{align*}
\]

### 6.4.3 Solving the Multi-Objective Optimization Model

To solve the multi-objective optimization model, we set different weights \(\omega_i\) to the construction costs \(C_d\) and the variance \(\sigma_i^2\) and change the multi-objective to the single objective optimization model.
Eventually, we obtain the model below:

$$\begin{align*}
\min & \omega_1 \sigma_i^2 + \omega_2 C_d \\
Q_t^i &= Q_0^i + \int_0^t q_{in}^i - q_{out}^i \, dt \\
C_d^i &= \alpha \cdot 0.34K_i^{0.9} \left[ \frac{V^i}{\left( \frac{H^i}{0.3} \right)^{0.74}} \right] \\
p^i &= \eta^i p q_{out} g h_t^i \\
q_{in}^{i+1} &= q_{out}^i \pm \Delta^i \\
0 &\leq q_{out}^i \leq q_{max}^i \\
Q_0^i &\leq Q_t^i \leq Q_{max}^i
\end{align*}$$

s. t.

- \sum_{i=1}^n p^i \geq P_K
- \sum_{i=1}^n Q_t^i \geq Q_K
- \omega_1 + \omega_2 = 1
- \omega_1, \omega_2 \geq 0

7 Results of Model

According to the Three-Dimensional topography of Zambezi River and requirement of geographical condition of building dams, we figure out the several possible locations of dams. Each dam will be estimated it’s own maximum water-storage: \( Q_{max}^i \).

![Figure 7 Flow chat of model solving](image)

Based on the model (6.4.3), we will figure out the optimum locations of dams by the flow chat of model solving Figure 7. Although the amount of possible results is \( \sum_{i=10}^{20} C_{20}^i \), we use the multi-level optimization search method to figure out the optimum locations of dams, which are shown below:
Figure 8 Locations of dams upstream

Figure 9 Locations of dams midstream

Figure 10 Locations of dams downstream
Table 1 Dams information

<table>
<thead>
<tr>
<th>Index</th>
<th>Name of Dam</th>
<th>Category</th>
<th>Project</th>
<th>Decimal degree latitude</th>
<th>Decimal degree longitude</th>
<th>Dam height (m)</th>
<th>Reservoir Capacity ($10^8 m^3$)</th>
<th>Hydroelectricity (MW)</th>
<th>Cost ($10^8$ $)</th>
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<td>1</td>
<td>Katombora</td>
<td>I</td>
<td>-17.84</td>
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8 The Strategies of Water Modulating

8.1 Water Modulating Model

Based on the Multi-Objective Optimization Model (6.4), we provide the governing equation as below:

\[ q_{\text{out}}^i = \frac{1}{\lambda^i} (q_{\text{in}}^i - q_{\text{AVG}}^i) + q_{\text{AVG}}^i \]

s. t. \( \lambda^i > 1 \)

Where:

- \( \lambda^i \) is governing equation coefficient of ith dam
- \( q_{\text{AVG}}^i \) is the average downstream water flow of the ith dam

8.2 The Curve of Flow Variation

We obtain the hydrological data of the Zambezi River Basin [3] and change discrete points into continuous curve. Data is approximated to the continuous curve, by using the piecewise linear interpolation method, as shown below:
8.3 The Lake Kariba’s capacity of water modulating with the Single-Dam Model

When $q_{in}^K > q_{AVG}^K$, $\lambda = 2$ ; $q_{in}^K < q_{AVG}^K$, $\lambda = 1.82$

8.3.1 In the regular situation

8.3.2 In the drought situation

8.3.3 In the flood situation
8.3.4 Conclusion

From Figure 12, 15, 17, we know that the Kariba Dam has good safety. Based on Figure 13, 14, 16, the Kariba Dam has a good capacity of water modulating, so the dam can give the Zambezi River Basin downstream a stable water flow continuously and eliminate the dry season even in extreme situation.

According to Figure 13, in the regular situation the peak flood will pass the dam two times per year which will cause serious damage to the dam. Besides, the Kariba Dam is useless to modulate the Zambezi River Basin upstream.

8.4 The Lake Kariba’s capacity of water modulating with the Multi-Dam System Model

8.4.1 In the regular situation

8.4.2 In the drought situation
8.4.3 In the flood situation

From Figure 12, 15, 17, 19, 21, 23, we know that the Multi-Dam System has a greater safety. According to Figure 13, 14, 16, 18, 20, 22, it is easy to find that the Lake Kariba will obtain a greater capacity of water modulating under the Multi-Dam System. Based on the results, the Multi-Dam System has a better capacity of water modulating, so the system can give the whole Zambezi River Basin a stable water flow continuously and eliminate the dry season even in extreme situation.
8.5 Strategy of water modulating

<table>
<thead>
<tr>
<th>Index(i)</th>
<th>Name of Dam</th>
<th>Definite condition of λ</th>
<th>λ</th>
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<tr>
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<td></td>
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<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Batoka</td>
<td>/</td>
<td>1</td>
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<tr>
<td>4</td>
<td>Devils</td>
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</tbody>
</table>

Table 2 Strategy of water modulating

9 Sensitive Analysis

9.1 Change the time of the flood peak

Conclusion: If the time of the flood peak change, the time when the flow reach the maximum after modulating will delay. As the Figure 25, the curve of water storage with the slight fluctuation will be tending stability.
9.2 Change the governing equation coefficient

![Figure 26 q-t](image1)

![Figure 27 Q-t](image2)

Conclusion: When the water inflow is steady, increasing the $\lambda$ will improve the capacity of water modulating of the Multi-Dam System and make the average water flow become stable. Based on the Figure 27, increasing the $\lambda$ will decrease the max value of water flow in the rainy season as to release the flood control pressure of dams downstream the Zambezi River Basin.

10 Strength & Weakness

10.1 Strength
1. We provide a reasonable and well-understood strategy to the ZRA managers.
2. Because of putting other water cycles into consideration, our model is close to reality.
3. Our model and control strategy can properly deal with emergency water flow situations.

10.2 Weakness
1. The hydrologic data of the Zambezi River Basin we used is accurate to a month, so the curve of flow variation is not so accurately.
2. The calculation of our model is complex, we may figure out the locally optimal solution by using the multi-level optimization search method.
3. We ignore the costs of maintaining dam after the dam has been built.
Reference


Appendix

1. Dam.m

```matlab
x=0:0.1:12;    % month
q=[96.3 79.5 110.5 274.6 795.3 1674.0 2304.1 2037.0 799.3 366.2 221.6 144.0 96.3]; % water flow
q1=[98.1 875.9 1047.9 1364.4 2436.8 1042.2 905.8 1162.7 1368.8 1111.2 1228.6 1075.5 981.9];
q2=[1277.7 1149.3 1296.6 1619.7 2741.1 1419.0 1455.0 1738.3 1888.8 1714.3 1632.0 1392.1 1277.7];

q1=[98.1 875.9 1047.9 1364.4 2436.8 1042.2 905.8 1162.7 1368.8 1111.2 1228.6 1075.5 981.9];
q2=[1277.7 1149.3 1296.6 1619.7 2741.1 1419.0 1455.0 1738.3 1888.8 1714.3 1632.0 1392.1 1277.7];
q=q2-q1;
figure(1)
plot(q1)
hold on
plot(q2)

E=sum(q)*0.1;    % total water per year
aver=E/12;    % water flow per month
aver_line=aver.*ones(1,length(x));
figure(2)
plot(x,aver_line);

q_con=Dam1(q,aver);
q_con=Dam4(q,aver);
plot(x,q_con);
hold on
plot(x,q_con,'r');

Q=Dam3(q,q_con,5.41e10);
plot(x,Q,'r');

Qmax=651e8.*ones(1,121);
Q0=433e8.*ones(1,121);
plot(x,Qmax,'r');
hold on
plot(x,Q0,'g');
hold on
```

2. Dam1.m
function q_con=Dam1(q,aver)
N=length(q);
q_con=zeros(1,N);
for i=1:N;
    if (q(i)>=aver)
        q_con(i)=0.645*(q(i)-aver)+aver;
    else
        q_con(i)=0.5815*(q(i)-aver)+aver;
    end
end

3. Dam2.m
function Q=Dam2(q,q_con,Q0)
N=length(q);
Q=zeros(1,N);
q_mix=q-q_con;
Q(1)=Q0;
for i=1:N-1
    Q(i+1)=Q(i)+(q_mix(i)+q_mix(i+1))*0.1/2;
end
end

4. Dam3.m
function Q=Dam3(q,q_con,Q0)
N=length(q);
Q=zeros(1,N);
q_mix=q-q_con;
Q(1)=Q0;
for i=1:N-1
    Q(i+1)=Q(i)+(q_mix(i)+q_mix(i+1))*0.1/2;
end
end

5. Dam4.m
function q_con=Dam4(q,aver)
N=length(q);
q_con=zeros(1,N);
for i=1:N;
    if (q(i)>=aver)
        q_con(i)=0.398*(q(i)-aver)+aver;
    else
        q_con(i)=0.2314*(q(i)-aver)+aver;
    end
end